**Analysis of Iterative Ensemble Smoothers for solving inverse problems.**

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*Sakov et al.* (2012) introduced the iterative EnKF (iEnKF) to improve the solution of sequential assimilation problems when the dynamical model and the measurement operator are highly nonlinear. The iEnKF solves the inverse problem for $\mathbf{x}_i$ given a forward model $\mathbf{g}(\mathbf{x})$ and observations $\mathbf{d}$,

$$\mathbf{x}_{i+1} = \mathbf{g}(\mathbf{x}_i), \qquad \mathbf{d} = \mathbf{h}(\mathbf{x}_{i+1}) + \mathbf{e}. \tag{1}$$

Since we assume the model to be perfect, we can compute the solution at time $\mathbf{x}_{i+1}$ as soon as we have updated $\mathbf{x}_i$ using the data at time $t_{i+1}$. The iEnKF reduces errors in the case with strong nonlinearities in the model or the measurement operator, compared to a non-iterative EnKF update.

The formulation above is identical to the inverse problem solved for parameter estimation in the reservoir engineering community except the model is written as a prediction $\mathbf{y}$ given a parametrization $\mathbf{x}$ and measurements of $\mathbf{y}$ (*Evensen*, 2018),

$$\mathbf{y} = \mathbf{g}(\mathbf{x}), \qquad \mathbf{d} = \mathbf{h}(\mathbf{y}) + \mathbf{e}. \tag{2}$$

The iterative ensemble smoother (IES) by *Chen and Oliver* (2012, 2013) and the ensemble smoother with multiple data assimilation (ESMDA) by *Emerick and Reynolds* (2012) are now becoming standard for parameter estimation in reservoir models. However, IES and ESMDA assume the model is perfect, and they update only the uncertain parameters before they rerun the ensemble simulation to obtain the final result. If we do not take significant model errors into account, there is a risk for an unphysical update of some uncertain parameters, that compensates for the neglected errors.

*Sakov et al.* (2018) extended the iEnKF to work for an imperfect model containing additive errors,

$$\mathbf{x}_{i+1} = \mathbf{g}(\mathbf{x}_i) + \mathbf{q}_{i+1}, \qquad \mathbf{d} = \mathbf{h}(\mathbf{x}_{i+1}) + \mathbf{e}. \tag{3}$$

We will discuss basic assumptions and properties of iterative ensemble methods for solving the inverse problem, and we will illustrate how they can reduce the impact of nonlinearity in ensemble methods. Further, we will explain how also IES and ESMDA can be reformulated to account for model errors and illustrate how mainly ESMDA is a viable alternative to the standard EnKF (and iEnKF) in the presence of strong nonlinearity.

## References

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